

A POSSIBLE IMPACT OF CALL OPTIONS FOR THE FREIGHT RATE ON THE INCOME IN CONTAINER SHIPPING

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Abstract

In this paper we consider impact of the simple european call options for the freight rate mainly on income of carriers, but also on transportation costs of shippers. Options are thought to be issued physically by a carrier as a guarantee for transporting the shipper's cargo at a specified future time for the rate not higher than the strike price. The option premium is calculated numerically based on the jump-diffusive model which was previously checked as the appropriate one in this market. We investigate several reasonable scenarios and tactics defined by such factors as option expiration time, permissible strike prices and intensity of agreements. The results are calculated using real historical data from one of market leading companies. They illustrate a hypothetical impact of options on income in the past assuming the company issued options for several recent years.

Key words: pricing and yield (revenue) management, liner shipping, jump-diffusion, freight rate call options.

1. Introduction

The Herfindahl-Hirschman Index, which is a measure of industry concentration, remains in the instance of the global container shipping market below 7% (see Alphaliner, 2012). Its values between 1% and 15% indicates an unconcentrated industry. It reflects properly the condition of the market where the top 10 carriers have together only about 60% of the market share. Thus, the market is rather competitive, in which the psychology of many independent competitors plays a crucial role and random models are a relevant tool for describing its behavior. Another consequence of this fact is a huge price volatility which is a great problem for both parties of the market — carriers and shippers. For the sake of simplicity the carriers may be understood as vessels owners, whereas the shippers as commodities owners (in fact the situation is more complex because of transport agents and vessel rentals). For instance, on the main trade route from South-East Asia to Europe the transportation prices have varied in the last 10 years from below USD 2000 up to over USD 4000.

As checked by Gardon (2014) in the case of the container shipping industry the price process may be properly modeled by a linear jump-diffusion driven by a homogeneous Poisson process. There are several conditions which must be fulfilled for a proper application of such a model, as a normal distribution of returns (called also ticks or relative process changes) except for jump times, the independence of the returns from preceding process values guaranteeing the linearity and the exponential distribution of an iid-sequence of interjump times consistent with the homogeneity of the driving Poisson process.

The model is defined by the following stochastic differential equation in the integral form:

$$\forall t \in (t_0, T] \quad X_t = X_{t_0} + \int_{t_0}^t aX_s ds + \int_{t_0^+}^t b\bar{X}_s dW_s + \int_{t_0^+}^t \bar{C}_s \bar{X}_s dN_s, \quad (1)$$

where the modeled process X denotes the weekly average net freight (or rate, which is a transportation price consisting of basic ocean freight and different surcharges like e.g. fuel surcharge) per transported unit (TEU or FFE - the volume of a 20 or 40 feet long standard container), $\bar{X}_t = X_{t-} = \lim_{s \nearrow t} X_s$, W is a standardized Wiener process, N is a homogeneous Poisson process with intensity λ and both driving processes are said to be independent. Coefficients a , b and C (\bar{C}_s means the left-hand side limit at s , as for X) are called here the drift, the volatility and the relative jump size, respectively. Further, C is a right continuous process constant between the jump (stopping) times (Γ_i) of the driving Poisson process and its values on the consecutive interjump intervals are realizations of an independent identically distributed (iid) sequence of random variables (\mathcal{C}_i), which may be treated as independent copies of a certain random variable \mathcal{C} , independent from the both driving processes W and N as well (see Mancini, 2009). Additionally, the sequence (τ_n) of stopping times will represent times when the process is observed. By \mathcal{F} we will denote the natural complete filtration generated by the relative jump size process and both driving processes.

Generally, it is difficult or even impossible to find a closed analytical formula for the solution of the jump-diffusion stochastic differential equation. Therefore, usually numerical methods are used for the approximation of the solution (see e.g. Gardoń, 2004, 2006). Fortunately, the explicit formula for the solution of the linear equation (1) may be derived using the generalized Itô formula for semimartingales (see Protter, 1990):

$$\forall t \in (t_0, T] \quad X_t = X_{t_0} e^{(a - \frac{b^2}{2})(t-t_0) + b(W_t - W_{t_0})} \prod_{i=N_{t_0}+1}^{N_t} (1 + \bar{C}_{\Gamma_i}). \quad (2)$$

The idea of freight options has been strongly considered in recent years (see Kou, 2002; Koekebakker et al., 2007; Nomikos et al., 2013), but the authors discussed usually exotic Asian options with a known distribution of \mathcal{C} . European Call options are a basic example of derivatives. In our case they give a shipper the right for buying an underlying shipping service at the fixed time (expiry date) in the future for a fixed price (strike price). For this right a shipper must pay a price called the option premium.

In this article we present the impact of the European Call options on the carrier's income. Firstly, we present the manners for estimation or calculation of all necessary variables. Further, we calibrate the model defined by (2) for the crucial route already mentioned from The Far East to Europe based on the historical data from an over 12 years long period. And finally, we investigate several reasonable scenarios and tactics of the option trading, defined by such factors as option expiration times, permissible strike prices and intensities of agreements. We check out how would the carrier's income have changed because of an additional gain or loss caused by options if the options had been issued for a certain part of containers shipped in the recent years. We discuss also in which sense such contracts may be valuable for shippers.

As mentioned in the beginning there is a great need for a hedging tool in the market because of huge freight fluctuations. Both shippers and carriers have had to face a strong uncertainty about the transportation cost or income in upcoming months. The parties have

tried to resolved the problem by informal forward contracts but they have failed when the freight had changed extremely until the cargo loading day. The forward contracts have been renegotiated in such instances, which has been insisted mainly by shippers in case of a freight drop but sometimes also by carriers when the freight had enormously increased. However, the problem could be managed by the options. A shipper buying the option pays for it immediately without the knowledge of future freight change. When it will increase over the strike the shipper will use the option and pay less than the market price. When conversely, the shipper will pay just the market price lower than expected, renouncing the useless option. But there will be no need or possibility for a renegotiation. From a carrier's point of view in case of a future freight decrease the carrier will earn an additional income from the useless option sold before. In the opposite case the carrier will be insisted by the contract paid many weeks or months ago (the option) on transporting shipper's commodities for a lower freight fixed by the strike, though, still satisfactory one, but in addition this will have been partially compensated by the premium gained before. The only question left unexplained is if shippers would be interested in paying the premium, more precisely, if they would not find it too expensive. But as we present further in the paper the premium is usually rather a tiny part of the transportation cost, therefore such a contract should be advantageous for both parties reducing the freight change risk for both contractors.

At the end of this section we want additionally to explain why European Put options, giving the buyer the right for selling (instead of buying) in the future a good or a service at the strike price, are not a natural instrument in this industry. It is obvious that a natural seller is of course a carrier offering the shipping possibility whereas a natural buyer is a shipper needing the commodities transportation service. Nevertheless, the European Put options might exist in a virtual sense as an additional hedging tool or a speculation instrument, analogously as in financial markets where an underlying asset is not sold physically. Unfortunately, such an approach requires strict legal regulations like those for stock exchange markets, whereas the European Call options physically realized might be issued by carriers just in the sense of a bilateral contract with a shipper.

2. Estimation and pricing

When applying the model (2) for the freight fluctuation all the parameters should be estimated, i.e. the drift a , the volatility b , the relative jump size C and the intensity λ of the driving Poisson process. Of course, the continuous model must be firstly discretized. A trajectory of the freight process X is observed at the stopping times (τ_n) and its relative changes are denoted by (Z_n) :

$$\forall n = 1, \dots, L \quad Z_n = \frac{\Delta X_n}{X_{\tau_{n-1}}} \approx \ln \frac{X_{\tau_n}}{X_{\tau_{n-1}}}, \quad (3)$$

where $\tau_0 = t_0$ and $\Delta X_n = X_{\tau_n} - X_{\tau_{n-1}}$ is the increment of the process X on the interval $(\tau_{n-1}, \tau_n]$. Firstly, for the proper evaluation of the risk the so-called no-arbitrage assumption is taken into account (see Kou, 2002). It eliminates a riskless opportunity for an almost sure (a.s.) relative gain greater than the risk-free rate existing in the market. An appropriate LIBOR serves usually as such a rate. Technically speaking the no-arbitrage assumption requires the model to be based on a stochastic process which is a local martingale with respect to a certain probability measure \mathbb{Q} ,

called the martingale measure or the pricing probability measure. As a consequence, this insists on taking a special estimate for the drift:

$$a = \rho - E\mathcal{C} , \quad (4)$$

where ρ is the LIBOR per time unit for 1 year USD contracts and E is the expectation operator. The expectation $E\mathcal{C}$ may be calculated from the theoretical distribution of \mathcal{C} or, if that is unknown as in our case, estimated by the sample arithmetic mean from the set of relative jump values.

Further, the volatility b is the infinitesimal variance of the continuous part (Z_n^c) of the returns (Z_n), that means the remained part after an extraction of jumps. Besides, this means that the standardized continuous part of the returns (Z_n^*) is a standard normally distributed iid-sequence:

$$\forall n = 1, \dots, L \quad Z_n^* = \frac{Z_n - a\Delta\tau_n - \sum_{i=N\tau_{n-1}+1}^{N\tau_n} \bar{C}_{\Gamma_i}}{b\sqrt{\Delta\tau_n}} \sim N(0, 1) . \quad (5)$$

Thus, the volatility b may be estimated by means of the maximal likelihood method as the standard deviation of (Z_n^c):

$$\hat{b} = \sqrt{\frac{1}{L} \sum_{n=1}^L \frac{(Z_n^c)^2}{\Delta\tau_n} - \left(\frac{1}{L} \sum_{n=1}^L \frac{Z_n^c}{\sqrt{\Delta\tau_n}} \right)^2} , \quad (6)$$

But for the proper estimation above firstly the jumps have to be separated from the continuous part of the data. There are several methods for jump identification. We decided for the so-called threshold method (see Mancini, 2009; Gardoń, 2011) with the threshold condition:

$$\frac{Z_n^2}{b^2} > r(\Delta\tau_n) , \quad r(t) = \beta t^{1-\varepsilon} \quad (7)$$

where r is the threshold function and usually $\varepsilon = 0.1$ and $\beta = \frac{2}{e\varepsilon} = 7.3576$. Each tick fulfilling the condition above is recognized as a jump. Already at the first sight the following trouble appears: in the threshold condition for jump identification the knowledge of b is necessary, but conversely, for the estimation of b the knowledge of jumps is necessary, as well. Fortunately, as shown by Gardoń (2011) it may be overcome by means of an iterative procedure. At the beginning b is estimated by (6) based on all the ticks and the first set of jumps is identified by means of the threshold condition (7). Then b is estimated again based on the returns except for the set of jumps found and the jumps are identified again. These steps are repeated until no new jumps occur. In this manner both b and a set of realizations of \mathcal{C} will be found. Later, the application of the bootstrap method to the set of relative jump sizes let us simulate the trajectories of the freight process X using the empirical distribution of jumps.

The last parameter left for estimating is the Poissonian intensity λ . If the process (2) is driven by the homogeneous Poisson process N with the intensity λ then the interjump periods should create an exponentially distributed iid-sequence with the same parameter as the Poissonian intensity. Therefore, we have decided to choose such one intensity which makes the exponential distribution fit the best to the empirical interjump periods in the sense of the maximal p-value of the Kolmogorov-Smirnov goodness-of-fit test.

The net premium O_t of the European Call option at time t , with the expiry time T and the strike price K is generally calculated (see Karatzas and Shreve, 1998) as the discounted expected value of the future profit with respect to the martingale probability measure \mathbb{Q} , i.e.:

$$\forall t \in (t_0, T] \quad O_t = \mathbb{E}^{\mathbb{Q}} \left(e^{-\rho(T-t)} \max\{X_T - K, 0\} \mid \mathcal{F}_t \right). \quad (8)$$

Sometimes, if the theoretical distribution of X is known, an explicit formula for the premium may be found, as in the case of the famous Black–Scholes formula where the underlying model is a geometric Brownian motion. But unfortunately, this is not our case because of the unknown distribution of the relative jump sizes \mathcal{C} . Of course, we could try to guess this distribution and test it but at the moment we decided to use the empirical one. Besides, even in the instance when the distribution of X is fully known it is not always a simple task to calculate the explicit analytical formula for O_t . Hence, we are not able to draw it here, though, the calculation of the premium is still possible by means of the Monte Carlo method. The details are described in the next section.

3. Simulations with the real data

Our aim has been to check the impact of the options sale on the income of a carrier based on the real historical data from one of the leading companies in the industry. More precisely, we have checked how would the carrier's income have changed because of an additional gain or loss from the European Call options if they had been issued for a certain part of containers shipped in the recent years. We considered several scenarios and tactics discussed separately in the subsections.

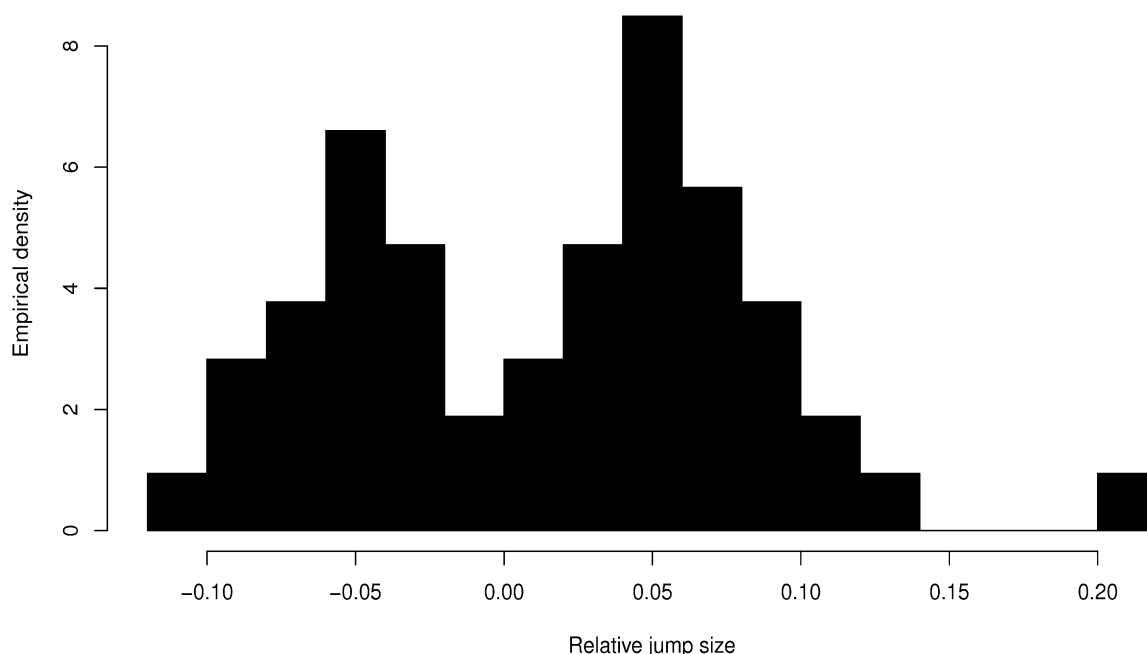
We have focused on the most essential and the most competitive trade route from South-East Asia to Europe, headhaul (more profit-yielding) direction. The data set consists of 657 weekly average net freight values and the same number of total weekly volumes from the over 12 years long time period, from January 2 2000 to August 5 2012. This produces $L = 656$ weekly returns of the freight process. As we mentioned before due to Gardoń (2014), the jump-diffusive model (2) fits well to the empirical data in this case.

The concrete estimates for the model parameters mentioned in the previous section are as follows. The volatility $b = 0.013$. The empirical distribution of relative jump sizes is shown in the Figure 1, with $\mathbb{E} \mathcal{C} = 0.0117$ estimated from 53 jump occurrences recognized by the threshold condition (7). On November 19 2012 corresponding to the data set the risk-free rate was equal to 0.86% p.a. due to the LIBOR for 1 year USD contracts, which implies $\rho = 0.0165\%$ per time unit which is a week (7 days). This is a common time unit in the entire industry. Poissonian intensity per time unit derived from the empirical distribution of the interjump periods is equal to $\lambda = 0.1923$. Summing up, the last parameter, namely the drift corresponding to the no-arbitrage assumption is $a = -0.0021$ in this case.

The option premiums were calculated based on $M = 10^5$ simulated trajectories of the Poisson process and the standard Brownian motion. For each trajectory N_T relative jump sizes were sampled from the empirical distribution presented in Figure 1. At first, just for the general information we want to discuss shortly the amount a shipper would have to pay in order to ensure the future transportation price at the reasonable level. The average net freight has been about 3000 USD/FFE, so let us treat it as a typical actual freight. Further, in our opinion, the most typical option should be the one with the strike price about this value and let

us take, for example, the expiry date half a year ahead. The net premium for this derivative is about 150 USD/FFE due to our calculations, i.e. about 5% of the freight. Stronger Calls, say with the strike price equal to 2700 USD/FFE, cost about 350 USD/FFE, whereas the weaker ones with the strike 3300 USD/FFE only about 50 USD/FFE. Hence, we concluded this would be an interesting and relatively cheap opportunity for shippers for limiting the possible future transportation expenses.

Figure 1: The empirical distribution of the relative jump size for the freight from South-East Asia to Europe headhaul direction based on 53 jumps recognized.



Source: the author.

Just for the comparison we want to add that the analogous premiums calculated based on the standard Black–Scholes method gave the values about 10–20% lower than the jump-diffusive model and even deeper differences in case of strikes differing strongly from an initial freight value. This is a well known problem that the Black–Scholes formula underestimates significantly such events which are far from the average. The fact is caused by the application of a pure Gaussian approach which underestimates the probability of outliers and which contradicts the heavy tails of the empirical data. Exactly as in our case where many outliers from the tails of the empirical returns distribution were extracted as jumps.

The net option premium is the so-called fair price for an option. That means in the infinite time horizon the accumulated profit (or loss) from the options sold will be equal to 0. Of course, within any finite time period random fluctuations may appear and the accumulated profit (or loss) may differ from the limit value. Among others for this reason the issuer should add a certain percentage of the net premium, a safety margin γ , to the final price. Let us assume that in this instance this safety margin is equal to $\gamma = 10\%$. It should not be significant for a buyer since, as mentioned already in this section, the net premium for the typical European Call is about 5% of the freight, now it rises only to about 5.5%. What is more, issuing of options generates usually some administrative costs, say ξ . Of course, they may be incorporated into

the safety margin but in case of a very cheap option the safety margin may be insufficient to cover such carrier's expenses. Let us assume $\xi = 10$ USD/FFE. For this reason we decided to consider in our scenarios the premium O_t^* including a commission in the form:

$$O_t^* = O_t + \max\{\gamma O_t, \xi\} \quad (9)$$

Another issue is that sometimes the freight drops so extremely, because of the strong competition in the market, that carriers are insisted on transporting the cargo under the profitability margin called the break-even. Let us denote it by B . We know this limit freight in case of the company which shared us the data for our disposal but we are not authorized to disclose it. Nevertheless, a carrier might not be eager to sell a Call when the strike price together with the premium is under this limit. For this reason in every scenario presented below we consider two tactics:

1. The options were issued unconditionally due to the scenario assumptions.
2. The options were sold only if $O_t^* + K \geq B$, where K is the corresponding strike price.

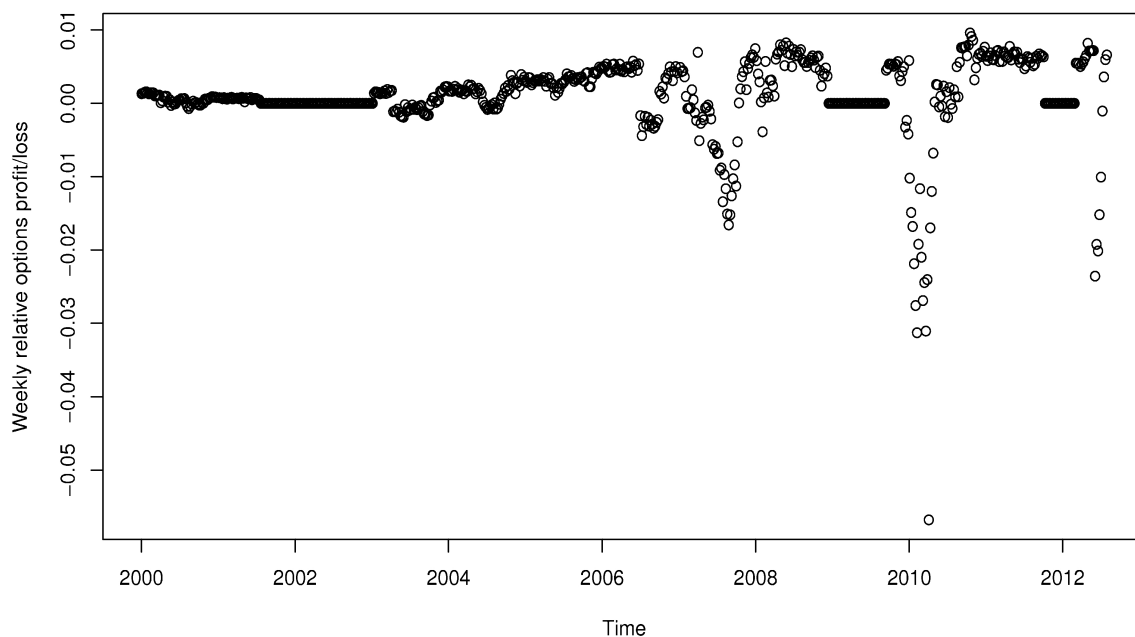
3.1 Scenario A

Let us begin with the simplest scenario fulfilling the following assumptions:

- Every week 10% of shipped containers were paid according to the European Call bought before by a shipper.
- All Calls were issued for the same time horizon of 3 months = 13 weeks (time units) ahead, i.e. $T - t = 13$.
- All Calls were issued with the strike price identical as the actual net freight, i.e. $K = X(t)$.

The assumptions are rather conservative, especially the last one concerning the strike price. We have also assumed a relatively short time horizon and a moderate demand on options.

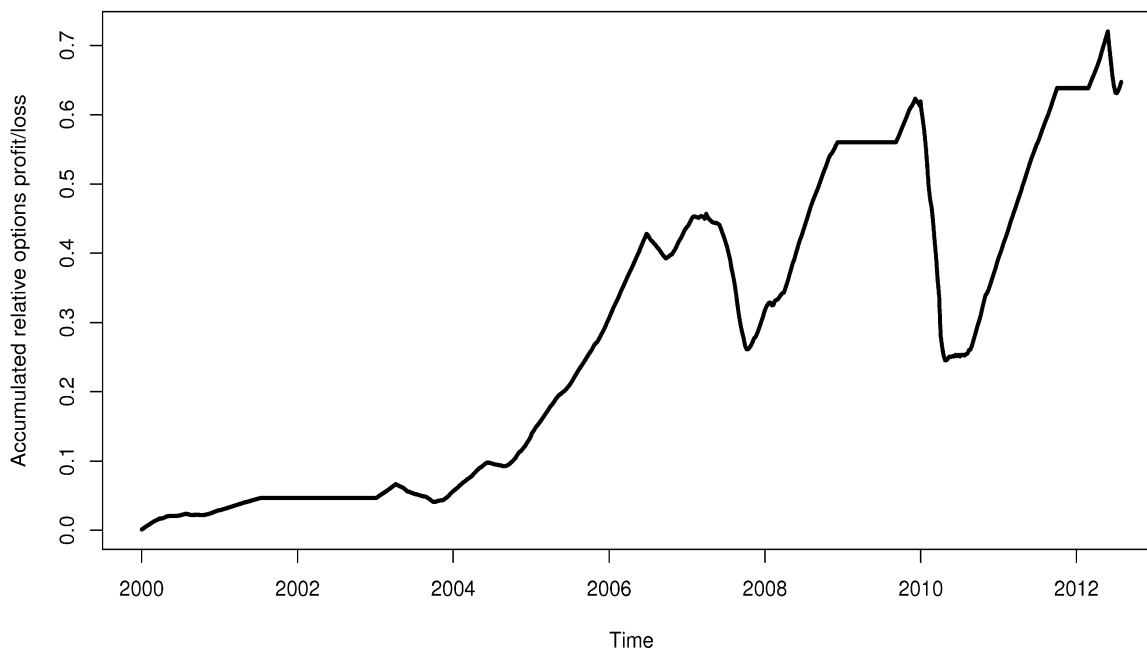
Figure 2: Weekly relative (to a yearly yield) options profit/loss due to scenario A, tactic 2.



Source: the author.

As we mentioned in the beginning of the section the net premium in a longer time horizon should give the accumulated profit approximately equal to 0. Therefore we conducted at first the standard T-test verifying if the final relative (regarding the total company yield in one of recent years) accumulated profit, equal in the scenario to -0.357 , differed significantly from 0. This is obviously equivalent to the test if the average weekly relative profit differed from 0. The p-value was equal to $p = 9.95\%$ which indicated it did not differ significantly from 0 at any reasonable significance level. Just for the comparison, an analogous relative accumulated profit calculated according to the standard Black–Scholes model was equal to -0.975 , with p-value $p = 0.001\%$ indicating clearly its negative value. This is the next signal that the Black–Scholes formula underestimates the derivatives. In addition we checked this property for the second tactic where the Calls would have been sold only in case the strike plus net premium had exceeded the break-even B (but without any commission). The p-value $p = 0.25\%$ suggested the relative accumulated options profit 0.437 was positive in this case. Of course, it was a consequence of the artificial elimination of such options which would have been completely disadvantageous for the carrier.

Figure 3: Accumulated relative (to a yearly yield) options profit/loss due to scenario A, tactic 2.



Source: the author.

As mentioned above in the scenario the additional profit from net premiums did not differ significantly from 0, though, it was negative. Unfortunately, it turned out that it remained still negative, equal to -0.124 , even according to the premium (9). This may be a warning for an issuer that the safety margin should be greater in this case than $\gamma = 10\%$. Nevertheless, even at 30% the premium would be usually an almost negligible part of the transportation price for a shipper. The situation looked much more optimistic in the second tactic. Of course, there were still weeks when the options sell would have led to a dramatic loss, as shown in Figure 2 where the week by week additional relative profit or loss is drawn, but the relative accumulated profit would have been positive during the entire time period. What is more, eventually, after over 12 years the option trading would have brought an additional profit equal to over 60% of the yearly yield which is presented in Figure 3. In both figures one can observe periods without any additional options profit. These were weeks when the sum of the premium and the strike were below the break-even B and the options trading would have been suspended due to the tactic.

The last comment to this scenario concerns the total profit volatility. We expected it would decrease because of an additional options profit in case of lower freight values and because of an options loss in case of higher freight values. And it dropped indeed but only slightly, from 0.4959 to 0.4943, so this effect of Calls is rather irrelevant, at least in this scenario.

3.2 Scenario B

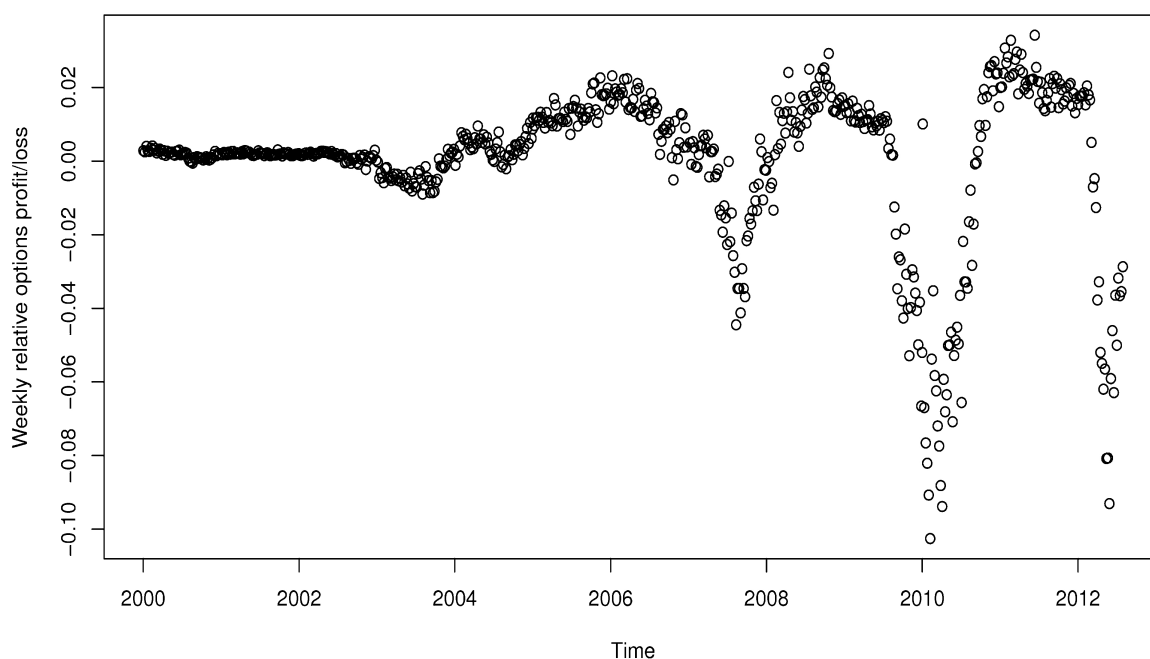
Now we investigate the options impact with more flexible assumptions:

- Every week a random ratio H_B of shipped containers was paid according to the European call bought before by a shipper, whereas $H_B \sim N(0.2, 0.03)$.

- 25% of all Calls were issued for the time horizon of 3 months = 13 time units ahead, 50% for the time horizon of 6 months = 26 time units ahead and 25% for the time horizon of 9 months = 39 time units ahead, i.e. $T - t = (13, 26, 39)$ with weights (25%, 50%, 25%).
- 10% of Calls for every time to expiration were issued with the strike price $0.9X(t)$, 20% with $0.95X(t)$, 40% with $X(t)$, 20% with $1.05X(t)$ and 10% with $1.1X(t)$, i.e. $K = (90\%, 95\%, 100\%, 105\%, 110\%)X(t)$ with weights (10%, 20%, 40%, 20%, 10%).

The scenario assumes twice as much Calls sold in average than the previous one. Besides, it takes into account also longer expiration times and more possibilities of strikes.

Figure 4: Weekly relative (to a yearly yield) options profit/loss due to scenario B, tactic 1.

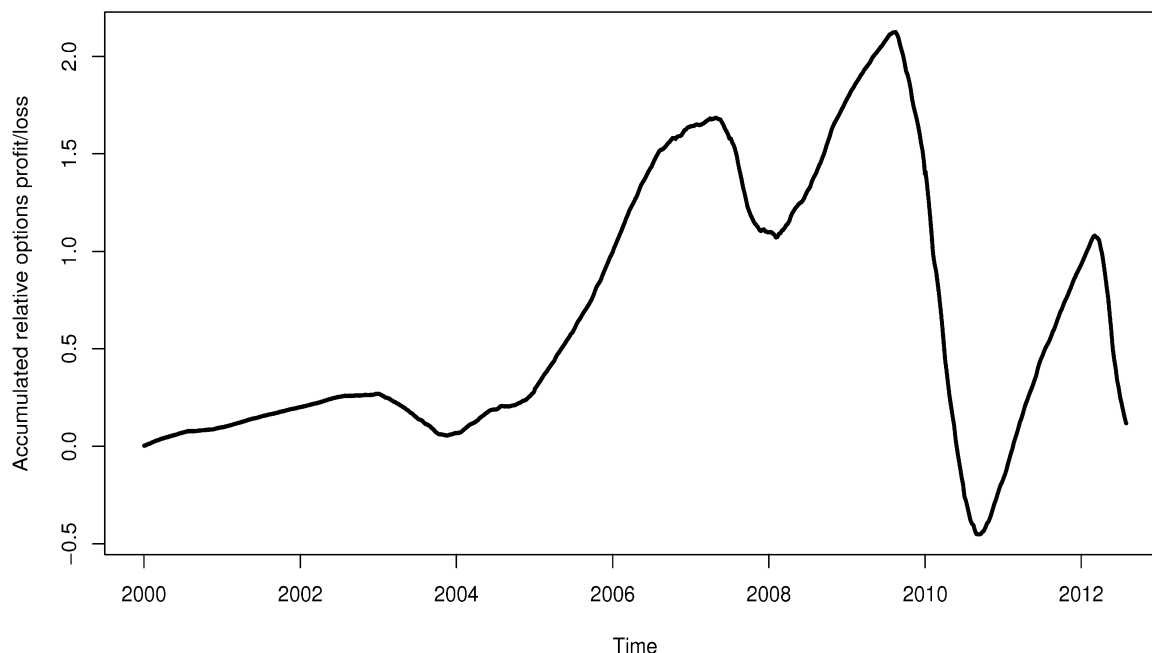


Source: the author.

The conclusions regarding the correctness of the model were similar as before. The accumulated relative profit from the net premium in the jump-diffusive model was equal in this instance to -0.718 differing insignificantly from 0 with really high p-value $p = 18.81\%$. Analogous comparison to the standard continuous model yielded the accumulated relative profit -3.419 , i.e. almost five times greater in the sense of the absolute value. Already at the first sight this was too huge difference to believe the nullhypothesis that it is zero, which was said to be definitely rejected by p-value $p = 2 \cdot 10^{-9}$.

In this instance already the first tactic with the premium (9) gave the positive additional profit of about 10% of the yearly yield which is presented in Figure 5. In Figure 4 there is shown additionally how this profit would have varied week by week. This is much better situation than in the previous scenario where an analogous result was negative. A possible explanation may be the greater volume of Calls issued in this case. Even much more optimistic conclusion may be drawn from the second tactic where the options profit was always positive, with a strongly increasing trend, finishing after over 12 years with a value exceeding 270% of the yearly yield. What is more, in this tactic already the net premium would have brought over 200% of the yearly yield which is, of course, significantly positive with p-value $p = 6 \cdot 10^{-11}$.

Figure 5: Accumulated relative (to a yearly yield) options profit/loss due to scenario B, tactic 1.



Source: the author.

The last comment concerns the volatility reduction which was slight again and dropped only from 0.4957 to 0.4909.

3.3 Scenario C

At last we check the results with fully random assumptions:

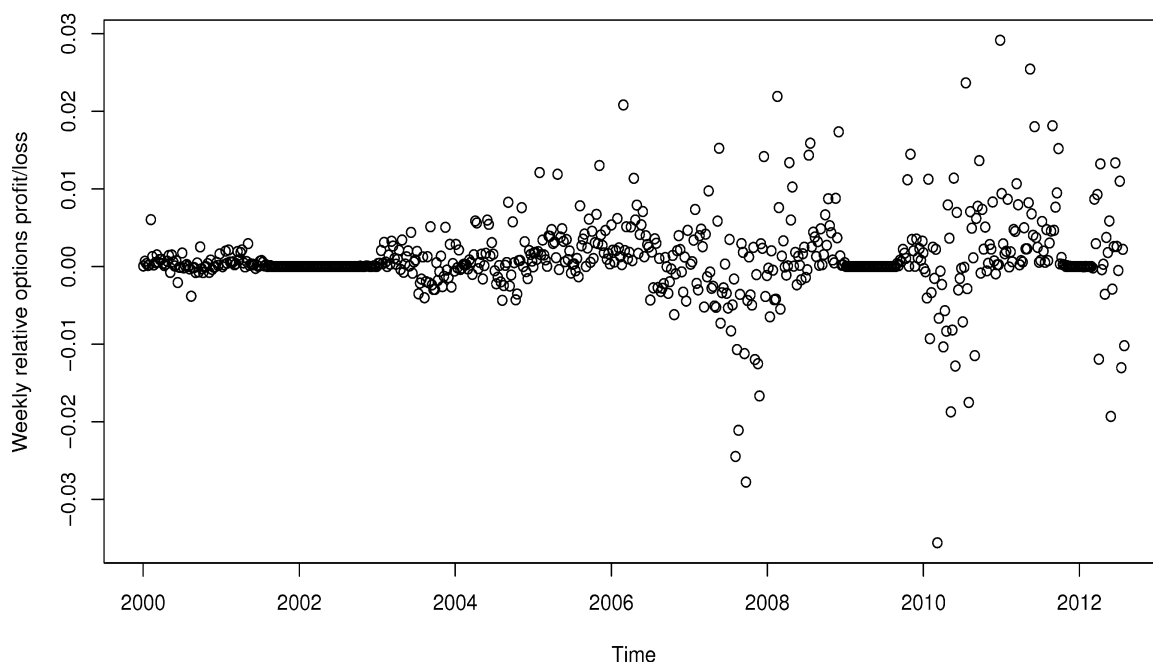
- Every week a random ratio H_C of shipped containers was paid according to the European Call bought before by a shipper, whereas $H_C \sim \text{Exp}(20)$.
- The Calls were issued for the time horizon of $T - t = (4, 13, 17, 22, 35, 52)$ time units ahead with random weights defined by the relative frequencies of a 10-element sample generated from the standard normal distribution, with the class limits $\{-2, -1, 0, 1, 2\}$.
- The Calls were issued with the strikes $K = X(t) + (-300, -200, -100, 0, 100, 200, 300)$ with the random weights defined by the relative frequencies of a 10-element sample generated from the $\Gamma(4, 20 \log 2)$ distribution, with class limits $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6\}$.

The amount of Calls issued weekly was the lowest this time in average since the expected value of an exponentially distributed random variable is the inverse of the parameter, equal here to only 5% of volume. The expiry dates were 1, 3, 4, 5, 8 or 12 months with random shares due to the standard normal distribution which were in average 2.3%, 13.6%, 34.1%, 34.1%, 13.6%, 2.3%, respectively. Similarly, the possible strikes were from USD $X(t) - 300$ to USD $X(t) + 300$, by USD 100, with random shares according to the $\Gamma(4, 20 \log 2)$ distribution which were in average 5.2%, 25%, 29.5%, 20.6%, 11.1%, 5.1%, 3.5%, respectively. In other words it was assumed in this instance that shippers would have rather preferred the strikes lower than the actual freight value. Generally, we thought the scenario might be adequate to the time

of first months or years after the options would appear in the market when shippers would show a kind of reserve to the new product.

We investigated identical characteristics as in both previous scenarios. The results describing the accuracy of the model confirmed our former conclusions. The model based on jump-diffusion gave the relative accumulated profit regarding the net premium equal to -0.274 , indistinguishable from 0 with p-value $p = 16.85\%$ and when applying the second tactic equal to 0.474 which was, as in other cases, significantly positive ($p = 0.02\%$). On the contrary, the Black–Scholes model yielded the significantly negative result -0.761 , with p-value $p = 0.008\%$.

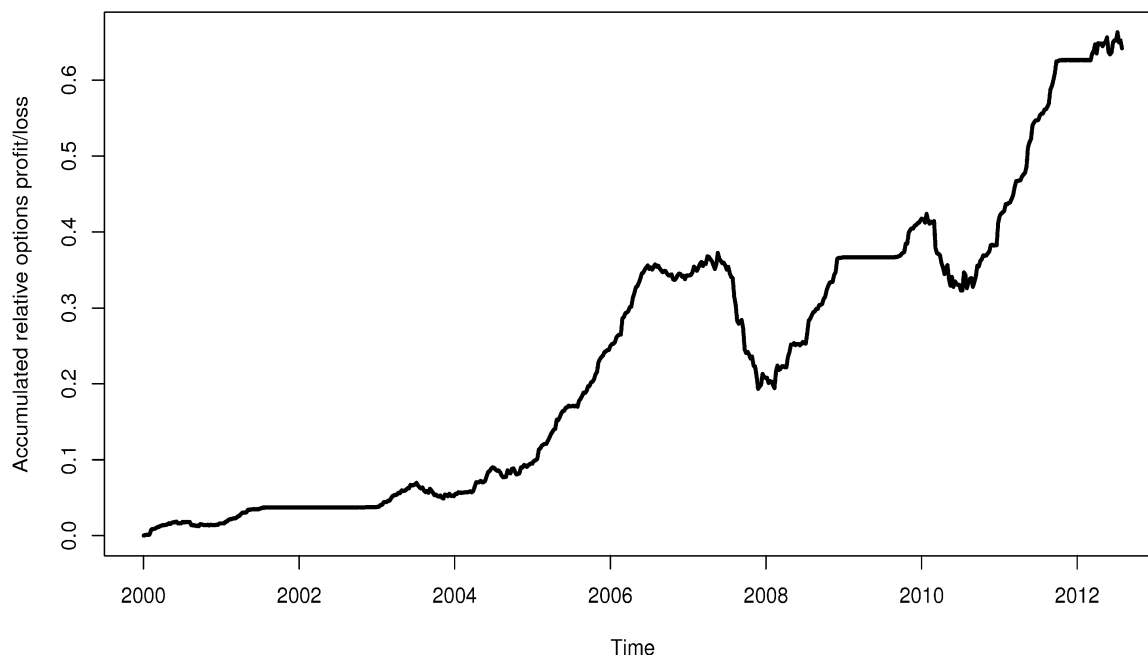
Figure 6: Weekly relative (to a yearly yield) options profit/loss due to scenario C, tactic 2.



Source: the author.

Similarly as in scenario A the first tactic, even together with the commission added to the net premium, gave a negative result, the total loss in amount of about 7% of the yearly yield. This convinced us more and more that the fraction of containers sold with options might be an important factor affecting the final success. There were no such worries in the second tactic. As we wrote several lines above even without any commission it yielded an additional profit. Together with the surcharges it gave the additional income higher than 60% of the yearly yield. The details of the effect of the tactic one can see in Figures 6 and 7. Again, there are such time periods visible when the Calls would not have been issued because of price levels which would have been unacceptable for carriers. And finally, identically as before only a tiny fall of the profit volatility was noticed in this case — from 0.4959 to 0.4946, surprisingly, completely inessential in the entire research.

Figure 7: Accumulated relative (to a yearly yield) options profit/loss due to scenario C, tactic 2.



Source: the author.

4. Conclusion

The experiments confirmed the adequacy of the jump-diffusive model in the container shipping industry. The profit based on the net premium in a longer time period did not statistically differ from 0 unlike in case of the standard Black–Sholes model. The idea of the simple European Call options for the net freight sold together with the contract for a cargo transportation service may deliver a carrier an additional revenue in the future. The amount depends apparently on the fraction of containers hedged by the option, the greater fraction the greater profit. But if this fraction is too low then a higher safety margin than 10% should be taken into account, added to the net premium as a commission. Otherwise it may lead to a significant loss. This problem may appear especially at the beginning of the options trading when shippers will not be familiar with the new product. At least in the first years from the time when options will have come into existence they shall be sold only if the joint profit of the premium and the strike price is over a satisfactory level called the break-even. This tactic led year by year in all simulations to the additional gain exceeding 5% of the yearly yield, in average. But opposite to the previous expectations the option trading did not reduce the volatility of the total weekly profit.

On the other hand, the Calls cost usually only a minor part of the freight even together with a surcharge. But at the same time they assure the upper bound for the transportation price paid by a shipper that limits its risk. Hence, they may be a reasonable solution for shippers in the problem of reducing the uncertainty about the future freight. Summing it up, this type of simply bilateral contract between a shipper and a carrier should be profitable for both parties.

Acknowledgements

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